Hausdorff dimension and filling factor

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Abstract

We propose a *new hierarchy scheme* for the *filling factor*, a parameter which characterizes the occurrence of the Fractional Quantum Hall Effect (FQHE). We consider the Hausdorff dimension, h, as a parameter for classifying fractional spin particles, such that, it is written in terms of the statistics of the collective excitations. The number h classifies these excitations with different statistics in terms of its homotopy class.

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In a series of papers [1], we have obtained a set of results with respect to fractional spin particles. Now we make the connection with the filling factor, a parameter that appears in the context of the FQHE and for it we have experimental values [2]. That connection can be made once the anyonic model has been considered to explain this phenomenon [3].

The FQHE is associated with a planar charged system in a perpendicular magnetic field such that a new type of correlated ground state occurs. The Hall resistance develops plateaus at quantized values in the vicinity of the filling factor or statistics, ν, which is related to the fraction of electrons that forms collective excitations as quasiholes or quasiparticles. Excitations above the Laughlin ground state are characterized, therefore, by ν and so we propose a *new hierarchy scheme* for the FQHE which gives us the possibility of *predicting* for which values of ν FQHE can be observed.

Our scheme is based on the intervals of definition of spin, s, for fractional spin particles which are related to the Hausdorff dimension, h. We verify that for some experimental values of ν for which the FQHE was observed the Hausdorff dimension is a rational number with an odd denominator (like the filling factor). Thus, bearing in mind the condition, $1 < h < 2$, we can determine for which values of h, the statistics give numbers in the intervals of definition, as follows:

$$\begin{array}{llll}
h_1 = 2-\nu, & 0<\nu<1; & h_2=\nu, & 1<\nu<2;\\
h_3 = 4-\nu, & 2<\nu<3; & h_4=\nu-2, & 3<\nu<4;\\
h_5 = 6-\nu, & 4<\nu<5; & h_6=\nu-4, & 5<\nu<6;\\
h_7 = 8-\nu, & 6<\nu<7; & h_8=\nu-6, & 7<\nu<8;\\
h_9 = 10-\nu, & 8<\nu<9; & h_{10}=\nu-8, & 9<\nu<10;\\
etc. & & &
\end{array} \tag{1}$$

In these formulas, h_i, represents the Hausdorff dimension of the collective excitations which are characterized by the statistics ν. This *hierarchy scheme* confirms our observation that*when the particles are interacting, in the presence of Chern-Simons field, the Hausdorff dimension changes because the statistics of the collective excitations change* [1]. Now, we give for some experimental values of ν the respective values of h, that is,

$$(h\ ,\ 0<\nu<1): \tag{2}$$
$$\left(\frac{9}{5},\frac{1}{5}\right),\ \left(\frac{12}{7},\frac{2}{7}\right),\ \left(\frac{5}{3},\frac{1}{3}\right),\ \left(\frac{8}{5},\frac{2}{5}\right),$$
$$\left(\frac{11}{7},\frac{3}{7}\right),\ \left(\frac{14}{9},\frac{4}{9}\right),\ \left(\frac{13}{9},\frac{5}{9}\right),\ \left(\frac{10}{7},\frac{4}{7}\right),$$
$$\left(\frac{7}{5},\frac{3}{5}\right),\ \left(\frac{4}{3},\frac{2}{3}\right),\ \left(\frac{6}{5},\frac{4}{5}\right);$$
$$(h\ ,\ 1<\nu<2): \tag{3}$$
$$\left(\frac{5}{3},\frac{5}{3}\right),\ \left(\frac{13}{7},\frac{13}{7}\right),\ \left(\frac{13}{9},\frac{13}{9}\right),$$
$$\left(\frac{10}{7},\frac{10}{7}\right),\ \left(\frac{7}{5},\frac{7}{5}\right),\ \left(\frac{4}{3},\frac{4}{3}\right);$$
$$(h\ ,\ 2<\nu<3): \tag{4}$$
$$\left(\frac{4}{3},\frac{8}{3}\right),\ \left(\frac{5}{3},\frac{7}{3}\right).$$

We can see another interesting point from these pairs of numbers: Some collective excitations with different spins have the same value of h, that is, the nature of the occurrence of FQHE for that values of ν can be classified in terms of h, so we can say that this number classifies *the collective excitations in terms of its homotopy class* [3]. In this way, the Laughlin wavefunctions [4] can be understood as a mapping between homotopy classes of the collective excitations.

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